ESM 204 Section Problems: Regulation over space and time

This exercise will compare three different regulatory approaches to controlling spatially dependent pollution: Uniform Roll-back, Emissions Permits, and Ambient Permit System

1. Suppose there are three firms emitting a spatially dependent pollutant. All firms initially emit 200 kg/ day of the pollutant. The regulatory agency is concerned with the ambient pollution levels at two different receptors. The table of transfer coefficients is given below:

Transfer Coefficients

Firm/Receptor	Receptor 1	Receptor 2
Firm 1	0.1	0.05
Firm 2	0.2	0.1
Firm 3	0.25	0.15

The background level of ambient pollution at each receptor is 15 kg.

What is the total concentration of the pollutant at receptor 1? Receptor 2?

PART 1: Uniform Roll-back

Now suppose the regulator is only concerned with pollution concentration at receptor 1. The regulator would like to achieve a total concentration of 100 (including background). What percent reduction in emissions must be achieved to reach this goal? What is the total cost of emissions reduction under the uniform roll back (each firm reduce emission by the percent reduction required to reach a concentration of 100) given the MC of reduction of each firm is given by $MC_1 = 10Q_1$, $MC_2 = 10Q_2$, and $MC_3 = 5Q_3$?

	With Background	Without
1	125	110
2	75	60

Receptor one will need to reduce concentration to 100, which implies that emissions need to be 73.72% (85/110) of the original level (or similarly reduced by 22.72% or 45.44 kg per firm) (Total reduction = 136.36).

Notice: When calculating percent reduction we use the desired concentration minus the background level. If the total emissions reaching the source equal 100, the total concentration at the receptor will be 115 due to the background.

TCi= ½* 45.44* (10*45.44) = 10,323.97

TC (1 and 2) = 2*10,323.97 = 20647,94

TC(3) = ½ *45.44 *(5*45.44) = 5161.98

TC^U= 25809.92

What is the resulting concentration of pollutant at receptor 1?
P1 = 154(.1 +.2+ .25) +1 5 = 99.7

PART 2: Emissions Permit System

Suppose the regulator instead decides to implement an emissions permit system. How many permits will be issued? What will be the price of each permit?

MC_1 (q1)=MC_2(q2) = MC_3(q3)
(i) 10q1=10q2=5q3

q1+q2+q3 = 136.36

Solving for q1 and q3 in terms of q2 using equation (i)…
q2 +q2 + 2q2 = 136.36
4q2= 136.36

q_2^E = 34.09
q_1^E =34.09
q_3^E =68.18
Total Emissions Permits = 600 – 34.09 – 34.09 – 68.18 = 463.64 permits issued
Price Emission Permits = MC(q) = $340.09

Total concentration at receptor 1 = 0.1(200-34.09) + 0.2(200-34.09) + 0.25(200-68.18) = 82.728 + 15 =97.72

TC1 = 5810.6405
TC2 = 5810.6408
TC3= 11621.28

TC^E = 23242.563

PART 3: Ambient Permit System

If instead the regulator implements an ambient pollution permit system, how many permits will be issued? What will be the price of each permit?

Spatial EM principal requires:

MC_1/a_1=MC_2/a_2 = MC_3/a_3
10q1/0.1 = 20q2/0.2 = 5q3/0.25
100q1 = 10q2 = 20q3

q1 = 0.5q2 =0.2q3

We must also reach the concentration target of 100 kg.

100 = a1e1 +a2e2 + a3e3 + B
100 = 0.1(200-q1) +0.2(200-q2) + 0.25(200-q3) + 15
85 = 110 – 0.1 q1 – 0.2q2 – 0.25q3
25 = 0.1q1 + 0.2q2 +0.25q3

Plugging in….

25 = 0.1(0.5q2) + 0.2q2 +0.25(2.5q2)
$\mathbf{q_2^A}$ = 28.57
$\mathbf{q_1^A}$ = 14.29
$\mathbf{q_3^A}$ = 71.43

Total Ambient Permits Issued= 600 – 28.57 – 14.29 – 71.43 = 485.71 permits issued
Price Emission Permits = $MC(q_1^A)/a_1 = MC(q_2^A)/a_2 = MC(q_3^A)/a_3$
= $1429.00

Total Cost (By firm)
TC1 = 0.5 * 10 * 14.29*14.29 = 1021.02
TC2 = 0.5 * 10 *28.57*28.58 = 4081.22
TC3 = 0.5 *5 * 71.43*71.43 = 12755.61225

TC^A = $17857.85

What is the least cost way of reaching a concentration level of 100 at receptor 1?

Ambient Permit System

2. Two identical firms save money from polluting. A firm's marginal savings from emitting an amount *e* are given by 24 – 3*e*. The two firms differ in their impact on ambient pollution concentrations. Three units of emissions from firm 1 result in one unit of ambient pollution. Firm 2 has three times the impact on the ambient environment from the same amount of emissions.
   MSi = 24 - 3e i = 1,3
   MD1 = 1/3 e
   MD2 = e

i. What are the transfer coefficients for the two firms?

$a_1 = 1/3$
$a_2 = 1$

ii. If firm 1 is given two emission permits and firm 2 is given six emission permits and they are allowed to trade, how many permits will each firm end up with and what will be the price?
There are multiple ways to solve this problem; here is one approach:

Initial allocation of emission permits:
2 permits to firm 1
6 permits to firm 2

Equimarginal constraint: $MS_1 = MS_2 \Rightarrow 24\text{-}3e_1 = 24\text{-}3e_2$
Allowance constraint: $e_1 + e_2 = 8$

$e_1 = e_2 = 4$ permits (for 1 unit of emissions each)
P=24-(3*4) = $12 per permit

iii. If instead each firm is given three ambient pollution permits and trading takes place, how much will each firm end up emitting and what will be the permit price?

Ambient permits: Firms will trade to the point where ambient permits, AP, have the same marginal value for both (i.e., the marginal cost, adjusted for the transfer coefficient, is equal). This is just the equimarginal constraint, where the firms are equating the marginal value of an ambient allowance rather than an emission allowance.

$MEB_1/a_1 = MEB_2/a_2$
$(24\text{-}3e_1)/a_1 = (24\text{-}3e_2)/a_2$
$(24\text{-}3e_1)/(1/3) = (24\text{-}3e_2)/(1)$
$(72\text{-}9e_1) = (24\text{-}3e_2)$

We also know that 6 total ambient permits are issued, so using the transfer coefficients:
$⅓e_1 + e_2 = 6$

Solving for e_1 and substituting back into the equimarginal constraint we derive:
$e_2 = 3.8$ permits
and, solving using the quantity constraint for e_1 we get:
$e_1 = 6.6$ permits 3

Check this answer against the equimarginal constraint:
$MEB_1/a_1 = MEB_2/a_2$
72-9(6.6) = 24-3(3.8) = 12.6 Checks!
This calculation also provides the price of the allowance, **P = $12.60/permit**

Check this answer against the equimarginal constraint:

$MEB_1/a_1 = MEB_2/a_2$

72-9(6.6) = 24-3(3.8) = 12.6 Checks!

This calculation also provides the price of the allowance, **P = $12.60/permit**